Invited Lecture

A Constructivist Approach towards Teaching and Learning Mathematics in Singapore: Rationale, Issues, and Challenges

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ABSTRACT. Enabling students to achieve a deep and connected understanding of mathematical concepts is an important aim in Singapore mathematics education. While current forms of instruction in the mathematics classroom can engender detailed expositions of a concept and links between targeted concepts and earlier concepts, much of this information is structured by the teacher and neglects the role of students' perspectives of the information that is transmitted to them. With the demonstrated efficacy of constructivist learning designs that build upon students' prior knowledge structures, one of such designs was implemented in Singapore's mathematics classrooms to not only afford deeper learning but also transform mathematics teaching and practice. In this paper, this constructivist learning design that was introduced to Singapore's secondary mathematics classroom is described and its rationale, efficacy, and the measures that were taken to ensure its sustainability discussed. The paper concludes with reflections of how to sustain such constructivist designs beyond research, and suggestions on proliferating their use among the Singapore teaching fraternity.

Keywords: Constructivist learning design; Deeper learning; Mathematics teaching and learning; Sustaining learning designs.

1. A Deeper Understanding of Mathematics: Potential of Constructivist Approaches

Getting students to develop deep and robust understanding of mathematics is a desired outcome of mathematics education, and this objective is emphasized in the recent updates of the Singapore's secondary mathematics curriculum (Grades 7 to 10; Ministry of Education, Singapore [MOE]: Curriculum Planning and Development Division [CPDD], 2019). To achieve this goal, it is essential that students are given the opportunities to explore the interconnected nature of mathematical concepts, which are "products of insight, logical reasoning and creative thinking" (MOE: CPDD, 2019, p. 5), and to participate in processes that afford the *active construction* of these

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interconnections. Unfortunately, pedagogical practices in the current Singapore mathematics classroom are largely didactic in nature (Kaur, 2009), and they may not be adequate in supporting students to engage in deeper learning. A paradigm shift is required, and one possible direction for this shift is for teachers to adopt pedagogies that are informed by constructivist learning perspectives.

Constructivism asserts that knowledge is *actively constructed* by the learner, and that knowledge is the product of one's cognitive acts via a meaning-making process (Applefield et al., 2001; Confrey, 1990a; Ertmer and Newby, 2013; Karagiorgi and Symeou, 2005). It posits that individual construction of knowledge can be influenced by the process of interaction and negotiation (Jaworski, 1994) with teachers (Green and Gedler, 2002; Vygotsky, 1978) and the learning context (Amineh and Asl, 2015). Such interaction and negotiation would enable teachers to clue into students' prior knowledge structures and knowledge construction processes during learning, and into the kinds of knowledge that they could build upon during instruction (Smith et al., 1993). Several learning designs like the "Open Ended Approach" (Becker and Shimada, 1997) and "Productive Failure" (Kapur, 2008, 2010), which involve the use of students' constructions in instruction, have been found to be efficacious in promoting student learning. This suggests that constructivist learning designs that build on students' constructions may have potentials to enhance students' connected understanding, which is one of the core objectives of the updated curriculum.

In this paper, a similar learning design — coined the *Constructivist Learning Design* (CLD) — that was developed by a team of mathematics researchers and educators in 2018, and later implemented in the Singapore secondary mathematics classrooms is described. The CLD's rationale, justification, and efficacy will be first examined, and this will be followed by an identification of the possible issues of sustaining such a design in the classrooms and a description of measures that were taken by the research team to address these issues. The paper closes with a reflection on the challenges of sustaining the use of CLD beyond the research project, with suggestions on propagating the pedagogical innovation among Singapore mathematics teachers.

2. The Constructivist Learning Design (CLD)

2.1. Engineering a constructivist learning environment in learning a new concept

The CLD is based on three propositions about learning from both cognitive constructivist (e.g., Confrey, 1990a; Noddings, 1990) and social constructivist positions (e.g., Brown et al., 1989; Savery and Duffy, 1995). First, constructivists posit that understanding is brought about through *an interaction between learners' prior conceptions and the context of learning.* This is supported by research on students' misconceptions and alternative conceptions (e.g., Confrey, 1990b), which showed that learners' prior conceptions, whether formal or informal, are activated and used as "resources" in the knowledge construction process. Acknowledging the importance of prior knowledge, constructivists suggest that students learn best in learning environment that allow for the compatibility of their prior constructions to be tested (Savery and Duffy, 1995), and this could be achieved through reflections and comparisons (Billing, 2007).

Second, learning is stimulated via *cognitive conflict or disequilibrium*, which determines the organisation and nature of what is learnt. What is "problematic" leads to and is the organiser for learning (Dewey, 1938; Roschelle, 1992), and this notion is also echoed in Piaget's (1970, 1977) theory of cognitive development, which maintains that knowledge construction is stimulated by internal cognitive conflict as learners strive to resolve mental disequilibrium (Applefield et al., 2001). Getting learners to realise gaps between their current knowledge and that of the targeted one also harmonises with Vygotsky's (1978) "zone of proximal development" (ZPD), where this "zone" illustrates the difference between what a student could achieve independently and what he or she could achieve with the guidance of knowledgeable others (e.g., teachers, peers).

Third, e*valuating the viability of individual understandings and social negotiation are important in the evolution of knowledge*. The catalyst for knowledge acquisition is via dialogue, and understanding is facilitated by exchanges that occur through social interaction, questioning and explaining, challenging, and offering timely support and feedback (Applefield et al., 2001). Meaning can be socially negotiated and understood based on viability (Savery and Duffy, 1995). Transfer is promoted when learning takes place through active engagement in social practices and is facilitated when learners are encouraged to talk about the similarity of representations for both the initial and targeted tasks (Billing, 2007).

From the propositions outlined above, four instructional principles that undergird the proposed CLD were outlined. These include (i) affording the elicitation and building upon of students' pre-existing informal or formal understanding of a concept, (ii) aiding the development of an organised and interconnected knowledge that facilitates retrieval and application, (iii) engaging students' thinking about their thinking and learning through conflict inducing processes, and (iv) building a social surround that allows for interpersonal and social nature of learning and this could be done via collaborative learning (see Lee et al., 2021 for more details). A viable learning approach that could fulfil the above principles also needs to be aligned to the school or national curriculum, and in the Singapore context, the CLD should be aligned with the updated secondary mathematical syllabus which emphasizes on *problem solving* that is central to the Singapore mathematics curriculum framework (MOE: CPDD, 2019). Given that, a possible way to support the generation of students' conceptions in the learning of new mathematical concepts could be through the introduction of a complex problem that targets a new concept that students had not been formally taught. This echoes the "teaching via problem solving" approach coined by Schroeder and Lester

(1989) and the "problem-solving first, instruction later" approach from the learning sciences (Loibl et al., 2017), both of which were pointed out by scholars as being suitable in introducing new mathematical ideas (e.g., Kapur and Bielaczyc, 2012; Nunokawa, 2000).

2.2. The CLD: description and justification

The considerations behind a viable constructivist learning design culminated in a two-phased CLD. The CLD comprised (i) a collaborative *problem-solving phase*, where students work collaboratively on a problem targeting a concept that students have yet to learn and attempt to generate innovative solutions to solve the problem, followed by (ii) an *instructional phase* where the teacher builds upon the solutions, creating linkages between the solutions to the targeted concepts.

2.2.1. CLD's problem solving phase

The problem in the problem-solving phase was designed such that it helps in the elicitation of students' prior knowledge structures. In line with other similar "problem-solving first, instruction later" learning designs (e.g., Becker and Shimada, 1997; Kapur and Bielaczyc, 2012), a complex problem targeting a concept or strategy is given to students to solve before the formal introduction of the concept or strategy. The problem, which contains different parameters, provides students' opportunities to tap on their intuitive or formal prior knowledge, and encourages the generation of multiple solutions. Past research suggests that while students were typically unable to generate or discover the correct solutions by themselves, they are able to generate a diverse set of solutions (e.g., Kapur, 2008; 2010; 2012; Kapur and Bielaczyc, 2012). The problem is also solved collaboratively, and such peer collaboration is necessary to allow for the negotiation of meaning of concepts (Lee et al., 2021). Beside keeping the groups on task and providing affective support to ensure that students persevere in solving the problem, the teacher also ensures that students experience conceptual conflict and disequilibrium. While refraining from telling students the solution to the problem, teacher facilitates students problem-solving efforts by pointing out their solutions' potential strengths and limitations and suggest ways to refine their strategies. The students' responses provide teacher with an insight into the gaps that are to be bridged between students' current conceptions and the targeted concept.

2.2.2. CLD's instruction phase

After the problem-solving phase, the teacher organizes students' solutions to the problem, and builds upon these to teach the targeted concept or strategy. The solutions are organised according to their relationship with the critical features of the targeted concept. The teacher then implements CLD's instruction phase, which aims to resolve the conceptual conflict and gaps that were induced during the problem-solving phase, effect the process of assimilation and accommodation (Piaget, 1977), to help students understand why the targeted concept or strategy is the most adaptable one given the problem. In line with past recommendations on how multiple solutions to problems could be consolidated (Kapur and Bielaczyc, 2012; Richland et al., 2017), the teacher discusses the affordances and constraints of each solution type, and compares and contrasts each solution's features with the critical features of the targeted concept, via counterexamples as much as possible. By getting students to consider the viability of their solutions vis-à-vis the targeted concept, the instruction phase acts as a platform for the negotiation and reflection of the concept's meaning, and therefore aids a deeper understanding. In addition, the CLD recommends the use of practice tasks that not only reinforce the procedural knowledge of the concept, but also further develop students' conceptual understanding. This is supported by Chinese Post Teahouse approach (Tan, 2013), a learning design which embodies constructivist principles, which advocates the use of appropriate practice tasks to complement the pertinent ideas that were brought up during instruction.

2.2.3. Justifying CLD: efficacy of similar learning designs

Past studies had shown that problem-centered learning designs that are similar to CLD were effective in helping students acquire better content and conceptual knowledge in K-12 classrooms and also in other settings like tertiary medical education and professional training development (e.g., Hung et al., 2008; Merritt et. al., 2017; Thomas, 2000). A review by Loibl et al. (2017) also demonstrated that a similar two-phased "problem-solving first, instruction later" instructional designs could potentially help to improve students' ability to transfer. For example, it was found that students who experienced the two-phased problem-first "productive failure" learning design significantly outperformed their counterparts in the traditional direct instruction condition on conceptual understanding and transfer problems without compromising on procedural fluency (e.g., Kapur, 2008, 2010, 2012; Kapur and Bielaczyc, 2012). Evidently, these positive findings provided justifications for the implementation of CLD in the actual ecologies of the classroom.

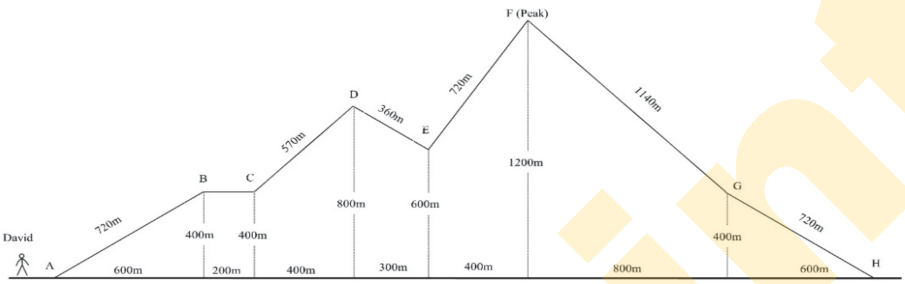
2.3. Implementing CLD and results

One of the CLD units that was designed targets the concept of gradient of linear graphs, which is introduced at secondary 1 (grade 7) level of the Singapore mathematics secondary level curriculum. The canonical gradient concept, which is a measure of steepness and direction of a straight line, is formulated as

$$\frac{\text{change in magnitude and direction of variable 1}}{\text{change in magnitude and direction of variable 2}} \text{ or } \frac{\text{Vertical change}}{\text{Horizontal change}} \text{ or } \frac{\text{Rise}}{\text{Run}}.$$

Together with a team of experienced Singapore mathematics educators, an analysis of the concept was conducted, and 4 critical features that underlie the gradient concept were identified: the (a) quantification/magnitude of steepness; (b) the quantification of direction; (c) the consideration of 2 dimensions/variables; and (d) the consideration of the ratio of 2 variables. Variations of these critical features were crafted within a plausible context, culminating in the complex problem task that we see in Fig. 1. In the problem task, students are asked to develop as many mathematical measures as they can to characterize the steepness and direction of 7 mountain trail sections, using given variables such as horizontal distances, absolute heights, and slope lengths.

The Mountain Trail



David enjoys hiking on a small mountain with a peak of 1200m. The figure above shows a sketch of David's trail, which has 7 sections. He starts at point A, hikes through points B to G, and ends at point H. The vertical heights, the horizontal distances, and the lengths of the slopes (rounded to the nearest 10m) are also indicated in the figure.

Although he is a seasoned hiker, David notices that some sections are steeper compared to others**.** He seeks your group's help to describe **both the steepness and direction of the mountain's slopes mathematically**. Here is what you must do:

(1) Assuming that all other things are equal, please use the information provided in the figure above and come up with as many ways as possible to rank the various sections of the trail both in terms of their steepness and the direction.
(2) For each way of ranking in (1), justify your ranking **mathematically** to describe both the steepness and direction of different sections.

All the best, and remember, don't give up until you have come up with as many methods as possible!

Fig. 1. "The Mountain Trail" problem of a Constructivist Learning Design (CLD) unit targeting the concept of gradient of linear graph.

The CLD unit on gradient of linear graphs was implemented to secondary 1 (Grade 7) students from various secondary schools. In a study that was conducted with students from a Singapore mainstream school (Lee et al., 2021), students in the CLD class spent a period (roughly 50 minutes) on the problem-solving phase and generated as many possible solutions as possible, while the instruction phase, which included teachers' consolidation of students' solutions and follow-up practice, took about four 45-minute periods. The CLD groups were able to produce an average of 4 solutions per group (*SD* = 1.95 solutions), and these solutions were categorized into 4 categories that were related to the critical features of the gradient concept, including those that considered (i) one dimension/variable, (ii) a combination of two dimensions/variables, (iii) a ratio of two dimensions/variables; and (iv) solutions that employed angles to determine the steepness and direction of the slopes (see more details of in Lee et al.

2021). During the consolidation of these solutions, the teacher invoked critical features of the gradient concept by actively comparing and contrasting each solution type relative to the others. For example, when analysing solutions that consider only the horizontal distance to measure steepness, the teacher noted that while such one-dimension measures are quantitative, they are insufficient, since slopes with the same horizontal distance have different steepness (e.g., comparing sections CD and EF in Fig. 1). In contrast, solutions with two dimensions might have better affordance.

To ascertain the efficacy and tractability of the CLD, the learning effects of the CLD was compared to its transmissionist, direct instruction (DI) counterpart on the learning of the concept of gradient of linear graphs (Lee et al., 2021). Students in the DI condition differed from the CLD condition in terms of the sequence of the problem solving and instruction phases. They first experienced the teacher-led instruction of the gradient of linear graphs concept guided by the course textbook, and after the instruction, worked on practice problems targeting the necessary procedural and conceptual understanding of the concept, from the same textbook that was used by their CLD counterparts. The teacher also went through the students' solutions, directing attention to the critical features of the targeted concept, and highlighted common errors and misconception. The learning outcomes of both conditions were compared via a 12-item post-test, comprising items assessing students' procedural knowledge, conceptual understanding, and ability to transfer knowledge of gradient to similar contexts (near transfer) and to more advanced concepts, such as gradient of curves (far transfer). Controlling for the effects of students' pre-requisite knowledge using a 4-item pre-test, a multivariate analysis of covariances revealed that the two learning conditions were significantly different in the learning outcomes, with subsequent tests of between-subject effects further indicating that the CLD class had significantly higher scores for conceptual understanding, near transfer, and far transfer (see Lee et al., 2021 for more details).

Like past "problem solving first" approaches, the CLD demonstrated the potential to develop more connected understanding of a concept. These findings provide a positive indication that the CLD has engendered deep learning processes to afford the cultivation of transferrable skills and knowledge (Lee et al., 2021). The demonstrated efficacy of the CLD unit paved the way for the development of more units that cover the major strands of the secondary mathematics syllabus, and these topics include angle properties of circles, standard deviation, and quadratic inequalities (see Ng et al., 2021).

3. CLD in Singapore Mathematics Classroom: Sustaining its Use

The demonstrated efficacy of the CLD and other similar constructivist pedagogical designs attests to their tractability in the mathematics classroom, and their potential in bringing transformative change in learning and teaching. Despite that, a major challenge is to ensure the *sustainability* of such learning designs in teacher practice, i.e., after the research, teachers are able to continue to employ these instructional innovations in the manner intended by its designers and make valid moves to own the designs, such that they become a part of their instructional repertoire (Coburn, 2003; Fishman, 2005). Teachers play critical roles in determining the degree of success in implementing instructional innovations (Doyle and Ponder, 1977; Ghaith and Yaghi, 1997; Guskey, 1987, 1988; Kennedy and Kennedy, 1996; Stein and Wang, 1988; Zhao et al., 2002). Given this, extant literature on possible factors that hinder teachers from sustaining instructional designs in their practice were surveyed and from which, programme and structures were developed to support teachers embracing a new instructional approach, i.e., CLD, in the classroom.

3.1. Teachers sustaining instructional innovations: knowledge, beliefs, and perceptions

A major reason why teachers do not actively use innovations in their instruction is a lack of teacher capacity (Ball et al., 2008, Shulman, 1986). Employing innovation places demands on teachers' content knowledge (CK) and pedagogical content knowledge (PCK). CK refers to teachers' subject-specific knowledge, while PCK is the subject matter knowledge unique to teaching, such as the knowledge of what makes certain topics easy or hard for students to grasp, of possible students' understandings and preconceptions, and of how to work with students' conceptions (Shulman, 1986). To design other topics using the learning design, teachers would also have to develop their design knowledge (DK) as well. Given these, efforts aiming to support teachers in sustaining the use of instructional innovations like CLD would do well to develop teachers' DK, CK, and PCK.

Teachers' expectations about learning and their perceptions of the utility of innovations also present challenges to continued use of an instructional innovation (Cohen and Ball, 1999; Fishman et al., 2011). Teachers' perceptions of their own students' abilities shape the kinds of instruction employed (Fishman et al. 2011), and their beliefs that certain strategies are more suited for their high achieving students than for the low-achieving ones, and vice versa, were documented in past research (e.g., Desimone et al., 2005; Stanovich, 1986; Young-Loveridge, 2005). With regard to teachers' perceptions of the usefulness of instructional innovations, past studies on various professional development (PD) programs showed that teachers' perceived coherence of the educational innovation with their personal goals, for learning and for their students, predicted their change in classroom practices (Garet et al., 2001; Penuel et al., 2007), and that having the capacity to make adaptations to the innovation is one of the key elements for long-term sustainability to occur (Shaharabani and Tal, 2017).

Taken together, for the CLD to be usable and sustainable for mathematics teachers, its alignment to teachers' knowledge, beliefs about learning, and their perceptions of its utility need to be considered. Given these, PD programmes and support structures were put in place to ensure the sustainability and relevance of the CLD in mathematics classrooms.

3.2. *Professional development*

To support teachers in implementing and sustaining constructivist learning designs in their practice, quality PD programmes are necessary to allow teachers to be conversant with implementing the necessary tasks and activity structures, and then progressing to eventually independently implementing, and possibly designing new units on their own. The PD principles that were adopted in the CLD research project were drawn from the continuous professional development model advocated by Fallik et al. (2008) and the PD model put forth by Markowitz and colleagues (2008). The *continuous professional development* model advocates the need for collaboration among teachers, partnership between teachers and facilitators, and support for teachers when they embark on any teaching method with their students. The need for teachers to be led through phases of being a learner, an instructor, and an innovator for any new pedagogical approach as they learn how to implement a new instructional method is recommended in Markowitz et al.'s (2008) model.

The recommendations made from these models were infused in the PD workshops and sessions designed for teachers prior to them implementing CLD in the classroom. These PD sessions were designed to not only enhance teachers' CK and PCK in implementing the unit, but also provide them with an embodied sense of what the CLD was from the standpoints of both a learner and teacher. After being introduced to the background, aims, and design principles of CLD, teachers then experienced a "problem-solving phase". Working in small groups, teachers examined a complex problem that were designed and evaluated students' representative solutions that were produced for each task. At the end of the evaluation, they were to provide a lesson plan of how they would build upon the students' solutions and instruct the targeted concept. In the 'instruction phase", a representative teacher from each group presented the group's solution. The trainer of the session, a research team member who is an academic faculty, Master Teacher, or Curriculum Specialist with experience in teacher training, consolidated teachers' responses and discussed ways to effectively compare and contrast student-generated solutions with the targeted concept. Teachers implementing the CLD units were also provided with detailed teacher's guides and with in-situ support by the research team during implementations.

3.3. *Networked learning community*

To support teachers', use of the CLD during and beyond the research, a Networked Learning Community (NLC) was also set up. Facilitated by both Master Teachers from the MOE's Academy of Singapore Teachers (AST) and a curriculum specialist, the NLC was set up with the aims to help build interested teachers' capacity to implement CLD, and champion its use in Singapore mathematics classrooms. Upon formation, the NLC organized meetings and workshops to equip a core group of interested teachers to deepen their CK and PCK in implementing existing CLD units, with sessions devoted to getting them to work with three chosen targeted concepts, consider their features, how they were being instructed in the curriculum, and students' prior knowledge and conceptions to these concepts. The NLC also enhanced teachers' DK in designing new CLD units. With support from Master Teachers and the curriculum specialist, the teachers worked collaboratively to decide on potential concepts that could be taught using the CLD approach, and then crafted the complex problems for the chosen concepts.

At present, the NLC has an active membership of 11 teachers from 7 schools. The NLC developed 2 units, and these units had at least one iteration in actual classrooms settings. In addition, the NLC also put in place structures to develop teachers who are relatively accomplished in implementing CLD to help seed the design through collegiate networks in their schools.

4. The CLD Beyond the Research: Challenges

The teacher capacity building efforts via PD programmes and resources, and the presence of the NLC platform to build a community of practitioners were measures that were put in place to ensure that a tractable alternative pedagogical approach like the CLD could be sustained during and beyond the research. The CLD's underlying principles are in line with curriculum's emphasis on deeper understanding of mathematics and problem-solving as focus. Such a research endeavor was made possible via a tripartite partnership among representatives from policy (MOE curriculum specialists), practitioner (Master Teachers from AST), and researchers from NIE. Nonetheless, ensuring a wider uptake of pedagogical innovations such as CLD remains a challenge in transforming Singapore education practice. As observed by Hung et al. (2022), the demands of a centralised Singapore education system that propelled Singapore's stellar performance in mathematics international assessments might explain the general inertia among mathematics practitioners in embracing innovations. Apart from identifying similar teacher capacity and beliefs issues brought up in the previous section, Hung et al. (2022) also noted institutional, policy, and cultural level issues behind the inertia. At the institutional level, teachers' educational background, and the lack of exposure to constructivist learning designs in both pre-service and in-service teacher programmes could explain teachers' lack of efficacy and unwillingness to implement such designs. At a policy level, while there is a push for such innovations in the classrooms, the high-stake assessments might disincentivise teachers in taking up instructional innovations that are perceived to be less efficient in getting students to master the necessary content knowledge. At a macro, cultural level, a (i) fear of failure that inhibits teachers' openness to unfamiliar instructional methods with unknown outcomes, and (ii) high power distance (Hofstede, 1991) that propagates the belief that knowledge provided by the teacher is absolute and final are possible inhibitors to pedagogical innovations that require some loosening of teachers' control in instruction.

Evidently, the factors identified by Hung et al. (2022) demonstrate the inextricable influence that the Singapore education system has on teachers in advancing and sustaining pedagogical innovations. To motivate change in teachers, there is a need of concerted actions from policymakers, researchers, and the teaching fraternity to further enhance the mathematics education for teachers to have the space and courage to implement pedagogical innovations independently. Drawing from the implications made by Hung et al. (2002) on how pedagogical innovations in Singapore could be advanced and the research team's experience with implementing and sustaining CLD in the Singapore mathematics classes, it seems that this tripartite synergy among policy makers, practitioners, and researchers is an important mechanism at addressing this gap, motivating a change of classroom culture and transforming practice. At the *policy* level, there could be a stipulation of the use of such innovations nationwide, a development of taxonomy that defines and operationalizes features of effective mathematics lessons, a provision of directives on the use of various assessment methods to assess mathematical competencies, and free up more space and time for teachers to implement these new pedagogies in the classroom. At the *practice* level, the presence of Professional Learning Communities and NLCs could not only help build teacher capacity, but also seed the innovation through its networks. These networks will be instrumental in developing teachers who champion and lead the innovation in their schools, effecting ecological leadership (e.g., Toh et al., 2016) and start micro-cultures that could shift socio-mathematical norms in the mathematics classrooms. As for the role of *research*, the research fraternity could work with policymakers and practitioners to develop the necessary resources in advancing these pedagogies; develop effective PD models that could equip Singapore teachers with the necessary capacities; adopt brokerage roles to understanding the needs of the ground and suggesting the necessary ideas and avenues for teachers to implement these strategies; embrace the essence of action research and teacher inquiry as measures of success of adaptation on the ground, and; continue their roles in helping policymakers and practitioners effect deeper understanding of mathematics.

5. Conclusion

In conclusion, the CLD was developed and put forth as a potential learning design that would promote deep and connected understanding of mathematics in students. As opposed to didactic forms of instruction, the CLD emphasizes more on the processes of problem solving to afford deep and meaningful learning and the development mathematical habits and dispositions in students. Moving beyond the research, it was postulated that cultural factors and teacher capacity were the reasons behind sluggish uptake of pedagogical innovation, and that these factors could be addressed by the concerted efforts to invest in teacher development, and push for a change in school and classroom culture. Furthermore, a tripartite synergy among policy, research, and practice could be important in sustaining new pedagogical approaches that have potential for deeper learning.

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